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PROBLEMS.

33. Proposed by F. P. MATZ, M. Sc., Ph. D., Professor of Mathematics and Astronomy in Irving College, Mechanicsburg, Pennsylvania

At what angle with the axis of a stalk must a *sharp* wedge-shaped blade be struck, in order to sever the stalk with the *least* force?

34. Proposed by O. W. ANTHONY, Professor of Mathematics and Astronomy, New Windsor College, New Windsor, Maryland

A particle is placed within a thin cylindrical shell without ends. Find the resultant attraction, the cylinder being composed of matter attracting according to the laws of nature.

AVERAGE AND PROBABILITY.

Conducted by B. F. FINKEL, Springfield, Mo. All contributions to this department should be sent to him.

SOLUTIONS OF PROBLEMS.

25. Proposed by G. B. M. ZERR, A. M., Ph. D., Vice President and Professor of Mathematics in Texarkana College, Texarkana, Arkansas.

The probability that the distance of two points taken at random in a given convex area, A , shall exceed a given limit (a) is

$$\Delta = \frac{1}{3A^2} \iint (C^3 - 3aC + 2a^3) dp d\theta,$$

where C is a chord of the area, whose co-ordinates are p, θ ; the integration extending to all values of p, θ , which give a chord $C > a$. What is Δ when the area is a circle? If in the circle $a = r = \text{radius}$, $\Delta = \frac{3\sqrt{3}}{4\pi}$.

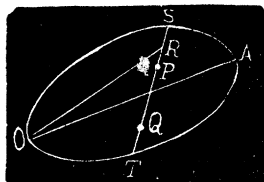
Solution by the PROPOSER.

Let ST be the chord; P, Q the random points, $OR = p$, the perpendicular, $\angle ROA = \theta$, $SQ = x$, $PQ = y$, $ST = C$.

An element of area at Q is $dp dx$; at P , $y d\theta dy$. The limits of x are 0 and $C - a$; of y , $C - x$ and a , and doubled.

Since two points can be taken in A^2 ways in the area A , we get for the required chance

$$\begin{aligned}
 \Delta &= \frac{2}{A^2} \iint d\theta dp \int_0^{C-a} \int_a^{C-x} y dx dy, \\
 &= \frac{1}{A^2} \iint d\theta dp \int_0^{C-a} (C^2 - 2Cx + x^2 - a^2) dx, \\
 &= \frac{1}{3A^2} \iint (C^3 - 3a^2 C + 2a^3) d\theta dp.
 \end{aligned}$$



Now let the area be a circle with the origin at centre. Then $C = 2\sqrt{r^2 - p^2}$, when r = radius. The limits of θ are 0 and $\frac{1}{2}\pi$, doubled, of p , 0 and $\frac{1}{2}\sqrt{4r^2 - a^2}$, and doubled.

$$\begin{aligned}
 \therefore \Delta &= \frac{4}{3\pi^2 r^4} \int_0^{\frac{1}{2}\pi} \int_0^{\frac{1}{2}\pi \sqrt{4r^2 - a^2}} \{ 8(r^2 - p^2)^{\frac{3}{2}} - 6a^2(r^2 - p^2)^{\frac{1}{2}} + 2a^3 \} d\theta dp, \\
 &= \frac{1}{2\pi^2 r^4} \left\{ (a^3 + 2ar^2)(4r^2 - a^2)^{\frac{3}{2}} + 8r^2(r^2 - a^2) \sin^{-1}\left(1 - \frac{a^2}{4r^2}\right)^{\frac{1}{2}} \right\} \int_0^{\frac{1}{2}\pi} d\theta \\
 &= \frac{1}{4\pi} \cdot \frac{a}{r} \left(2 + \frac{a^2}{r^2}\right) \left(4 - \frac{a^2}{r^2}\right)^{\frac{1}{2}} + \frac{2}{\pi} \left(1 - \frac{a^2}{r^2}\right) \sin^{-1}\left(1 - \frac{a^2}{4r^2}\right)^{\frac{1}{2}}.
 \end{aligned}$$

$$\text{If } a=r, \Delta = \frac{31}{4\pi}.$$

26. Proposed by J. WATSON, Middlecreek, Ohio.

Find the average area of all right-angled triangles having a given hypotenuse.

I. Solution by F. P. MATZ, M. Sc., Ph. D., Professor of Mathematics and Astronomy in Irving College, Mechanicsburg, Pennsylvania.

Let h = the given hypotenuse, and x = the base; then will $\sqrt{h^2 - x^2}$ = the perpendicular, and the area of the triangle is $A = \frac{1}{2}x\sqrt{h^2 - x^2}$. Hence the required average area becomes, if $\frac{1}{2}h\sqrt{2} = a$,

$$A = \int_0^a A dx \div \int_0^a dx, = \frac{1}{2}h^2(2\sqrt{2}-1).$$

Second Solution.

Represent the base by $h \cos \theta$, and the perpendicular by $h \sin \theta$; then

$$\text{we have } A = \frac{1}{2}h^2 \int_0^{\frac{1}{2}\pi} \sin \theta \cos^2 \theta d\theta \div \int_0^{\frac{1}{2}\pi} \cos \theta d\theta, = \frac{1}{2}h^2(2\sqrt{2}-1).$$

II. Solution by G. B. M. ZERR, A. M., Ph. D., Vice President and Professor of Mathematics in Texarkana College, Texarkana, Arkansas; O. W. ANTHONY, Professor of Mathematics, New Windsor College, New Windsor, Maryland; J. F. W. SCHEFFER, A. M., Hagerstown, Maryland; and H. W. DRAUGHON, Ohio, Mississippi.

Let $AC = 2a$ = hypotenuse of triangle, $AD = DC = DB = a$, and $\angle CDB = \theta$. $\therefore BE = a \sin \theta$.